PD-253

(512) M.A./M.Sc. MATHEMATICS (SECOND SEMESTER)

Examination JUNE 2021

Paper-I

Name/Title of Paper- ADVANCE ABSTRACT ALGEBRA (II)

Time: 3:00 Hrs.]

[Maximum Marks: 80

[Minimum Pass Marks: ---

Note: Answer from Both the Section as Directed. The Figures in the right-hand margin indicate marks.

Section(A)

Q.1.

1. Give example of a subring of Notherian ring which is not Notherian.
2. Construct Splitting field over Q for $(x^6\text{-}1)$
3. Every finitely generated module is a homomorphic image of____________
4. Defien primitive polynomial with example.
5. State Knonecker theorem.
6. Write the algebraic closure of R.
7. Is $R\sqrt{-5}\ normal\ over\ R$?
8. Find [C:R] C set of complex numbers and R set of real numbers.
9. State fundamental theorem of algebra.
10. Define U-Primary Module.

Q.2.

a) Define finilelty generated modules.

b) Deine reducibility of a polynomial is any field.

c) Discuss the reducibility of $x^4 + 8 \in Q(x), over\ Q$.

d) Find the degree of $Q(\sqrt{2}, \sqrt{3})\ over\ Q$.

e) Define normal extension with example.

Section(B)

Q.3.

a) Prove that in any finite field any element can be written as the Sum of two Squares.

b) Find θ, Such that $Q(\sqrt{2}, \sqrt{3}) = Q(\theta)$

OR

a. Prove that a finfite extension of a finite field is separable.

b. Let R be a noetherian integral domain then show that for all $C \neq 0, C \in R, Rc\ is\ large.$

Q.4.

a) Let F be a field of Characteristic $\neq 2. let\ x^2 - a \in$ $F[x]\ \ be\ an\ irreducible\ polynimial\ ov\ er\ F\ then\ find\ its\ Galoes\ group, what\ is\ the\ order\ of$ Galois group .

b) Find Conditions on a and b such that the splitting field of $x^3 + ax + b \in Q(x)$ *has degree of extension* 3 *over* Q.

OR

a. Find dimension of Q(a) over Q(b) Where $a = e^{\frac{2\pi}{n}i}$ *and* b *real part of* $e^{\frac{2\pi}{n}i}$.

b. Show that every principal left ideal in an integral domain R with unity is free as a left R-Module

Q.5.

a) Prove that Hom,(Q,Q) $=^N$ Q.

b) Find generators for the multiplicative groups of field with 13 elements.

Or

a. Find the galoes group of $x^4 - 3 \in Q(x)$.

b. Give example of a ring which is not left noetherian but right noetherian with explaination.

Q.6.

a) Show that $V = \oplus \sum_{i=1}^{3} \text{Rxi}$ where $x_1 = (1,0,0), x_{2=} = (1,1,0), x_3 = (1,1,1)$ and $V = R^3$ is a vector spvace over R.

b) Find subgroup of order 4 and their fixed field if E is the splitting field of $x^4 - 2 \in Q(x)$ *over* Q.

Q.7.

a) Show that R is artinian but not noetherian.

b) Find the Smallest extension of Q having a root of $x^4 - 5 \in Q(x)$

OR

a. Show that $\frac{Z}{(P,m)}$ *is a completely reducible* $Z -$ *module for two distinct primes* P *and* m

b. Find rank of linear mapping $Q: R^4 \rightarrow R^3$ *if* $\emptyset(a, b, c, d) = (2a - b + 3c + d, a - 8b + 6c + 8d, a + 2b - 2d))$